→ Συνέπεια Βασικών Εξισώσεων.

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Θ.Ε.Μ

→ $\tan x = a$

Βρίσκουμε ένα θ ώστε $\tan\theta = a$ (Πάντα υπάρχει)

Η εξίσωση γράφεται $\tan x = \tan\theta \Leftrightarrow$

$x = \kappa\pi + \theta \ , \kappa \in \mathbb{Z}$

→ Ομοίως $\cot x = a$

→ $\cos(a-b) = \underline{\cos a \cdot \cos b + \sin a \cdot \sin b}$ (1)

$\cos(a+b) = \cos[a-(-b)] = \cos a \cdot \cos(-b) + \sin a \cdot \sin(-b) =$

$= \cos(a) \cdot \cos(b) + \sin a \cdot (-\sin b)$

$\underline{= \cos(a) \cdot \cos(b) - \sin(a) \cdot \sin(b)}$ (2)

→ $\sin(a+b) = \cos\left[\frac{\pi}{2} - (a+b)\right] = \cos\left[\left(\frac{\pi}{2} - a\right) - b\right] =$

$\cos\left(\frac{\pi}{2} - a\right) \cdot \cos(b) + \sin\left(\frac{\pi}{2} - a\right) \cdot \sin(b) =$

$\underline{\sin(a) \cdot \cos(b) + \cos(a) \cdot \sin(b)}$ (3)

→ $\sin(a-b) = \sin[a+(-b)] \overset{③}{=} \sin(a) \cdot \cos(-b) + \cos(a) \cdot \sin(-b) =$

$\sin(a) \cdot \cos(b) + \cos(a)(-\sin b) = \underline{\sin a \cdot \cos b - \cos a \cdot \sin b}$ (4)

→ $\tan(a+b) = \dfrac{\sin(a+b)}{\cos(a+b)} = \dfrac{\sin a \cdot \cos b + \cos a \cdot \sin b}{\sin a \cdot \sin b + \cos a \cdot \cos b} = \dfrac{\frac{\sin a \cdot \cos b}{\cos a \cos b} + \frac{\sin b \cos a}{\cos a \cos b}}{\frac{\sin a \sin b}{\cos a \cos b} + \frac{\cos a \cos b}{\cos a \cos b}} -$

$= \dfrac{\tan a + \tan b}{\underline{1 + \tan a \cdot \tan b}}$ (5)

Τυποι διπλασιου τόξου

$\rightarrow \sin(2a) = \sin(a+a) = \sin a \cos a + \sin a \cos a = 2 \sin a \cdot \cos a$

$$\sin 2a = 2 \sin a \cdot \cos a$$

$\rightarrow \cos(2a) = \cos(a+a) = \sin a \cdot \sin a + \cos a \cdot \cos a = \sin^2 a + \cos^2 a \left.\begin{matrix} ? \\ \cos^2 a + \sin^2 a = 1 \end{matrix}\right\} =$

$= (1 - \sin^2 a) - \sin^2 a$

$$\cos(2a) = 1 - 2\sin^2 a$$

Ή

$$\cos(2a) = 2\cos^2 a - 1$$

Τυποι αποτετραγωνισμού

$\cos(2a) = 1 - 2\sin^2 a \Rightarrow$

$2\sin^2 a = 1 - \cos(2a) \Rightarrow$ $\boxed{\sin^2 a = \dfrac{1 - \cos(2a)}{2}}$

Ομοίως:

$$\boxed{\cos^2(a) = \frac{1 + \cos(2a)}{2}}$$

Μετατροπή γινομένων σε αθροίσματα

$\rightarrow \sin(a+\beta) = \sin a \cos\beta + \sin\beta \cos a$

$\sin(a-\beta) = \sin a \cos\beta - \sin\beta \cos a$

Προσθέτονται κατα μέλη

$\sin(a+\beta) + \sin(a-\beta) = 2\sin a \cos\beta \Rightarrow$

$$2\sin a \cdot \cos\beta = \sin(a+\beta) + \sin(a-\beta)$$